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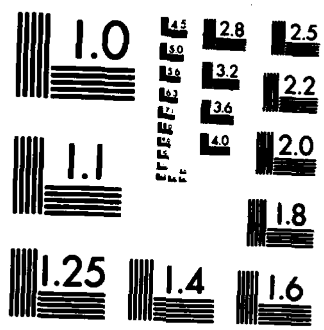

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FRONTOGENESIS IN AN ADVECTIVE MIXED LAYER MODEL

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ABSTRACT

Through an analysis of a more dimensional extension of the bulk mixed layer model of Kraus and Turner (1967) it is shown how even under spatially uniform atmospheric conditions an initially smooth horizontal temperature gradient in the surface mixed layer can still develop into a front. Essential for it is the notion that the net downward heat flux at the air-water interface is related to the difference between the mixed layer temperature T and an apparent atmospheric temperature T_A (Haney, 1971) so that the initial horizontal gradient in T corresponds to a horizontal variation in the surface buoyancy flux. As a result also the mixed layer depth differs from place to place. Depending on the direction of the wind driven transport this produces either a steepening or a flattening of the initial temperature profile. A lower bound condition for the initial horizontal advective heat flux is derived in terms of the initial surface heat flux, the stirring by the wind and the time rate of change of the apparent atmospheric temperature. If that condition is satisfied a front develops. If not, then frontogenesis is prevented by the damping effect of the local atmospheric heating. It is shown that the condition can be satisfied on the scales of lakes (or small seas) and in near equatorial oceanic regions. Mathematically the problem boils down to a first order quasilinear hyperbolic partial differential equation that is solved exactly by the method of characteristics.

AMS (MOS) Subject Classifications: 35L67, 76L05, 86A05

Key Words: frontogenesis, mixed layer dynamics, air-sea interaction, quasilinear heat equation

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FRONTOGENESIS IN AN ADVECTIVE MIXED LAYER MODEL

Will P. M. de Ruijter*

1. Introduction

Until rather recently bulk models developed for the upper mixed layer of the sea were mainly one-dimensional (see e.g. the review by Niiler and Kraus, 1977). This was motivated, among other things, by studies like the one by Gill and Niiler (1973) who used rough scaling arguments to show that advection plays only a minor role in the evolution of the mixed layer in most oceanic regions not too close to the equator. Near the equator wind-driven transports in the upper layer can be considerably larger so that horizontal advection can play a more prominent role in the mixed layer dynamics of those regions. Also in stratified lakes and reservoirs, with their completely different scales, river throughflows, selective withdrawal and intrusions, possible boundary and bottom effects etc. (see the review by Imberger and Hamblin, 1982) advection can play a dominant role.

One of the most dramatic effects of horizontal advection can be the formation of fronts in the upper layer and it is mainly in this context that the one-dimensional bulk mixed layer models were recently extended to the more dimensional case. Examples are the steady state model of Welander (1981) and studies by De Szoeke (1980) and Cushman-Roisin (1981). In all these models the physical process that is responsible for the generation of fronts is the horizontal variability of the wind stress such that it produces a convergent Ekman transport.

In this study we will isolate a completely different physical mechanism that can produce frontogenesis. It does not depend on a horizontal gradient in either wind stress

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or atmospheric temperature at the surface: both shall be spatially uniform throughout this paper. An essential ingredient for it is that the downward heat flux at the surface is not merely a prescribed function but related to the difference between the mixed layer temperature and an apparent atmospheric equilibrium temperature T_A that includes the effects of evaporation and solar radiation (Haney, 1971). An initially smooth, horizontal gradient of the mixed layer temperature thus produces a variability in the surface buoyancy flux. Associated with that also the mixed layer depth shows a horizontal variation. With a uniform wind-driven transport this can under certain conditions (derived in Section 3) lead to a steepening of the temperature profile with eventually, in the absence of a horizontally diffusive process, the formation of a discontinuous step in the temperature distribution.

In Section 2 we present the basic mathematical formulation of the model which is in essence the more dimensional extension of the classical model by Kraus and Turner (1967). In Section 3 we then show that for the non-entraining application the process can be described by a first order quasilinear hyperbolic partial differential equation that can be solved analytically by classical methods. *Inspiration for* its analysis (Section 4) can be based on the study of related problems in completely different fields of application ranging from shock waves in gas dynamics (Courant and Friedrichs, 1948) to the evolution of jams in traffic flow (Lighthill and Whitham, 1955; Richards, 1956). Many of those specific applications are described in Whitham (1974).

2. Formulation of the model

In this section the mathematical formulation of the advective mixed layer model shall be presented. The starting point is the heat conservation equation, in which horizontal diffusion is omitted:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} + \frac{\partial}{\partial z}\overline{w'T'} = -\frac{1}{\rho_0 c}\frac{\partial I}{\partial z} . \qquad (1)$$

T is the bulk temperature; u, v and w are the velocity components along the x, y and z axes, respectively; x is positive eastward, the positive y axis points northward and z is positive upward from the watersurface $z = 0$; $\overline{w'T'}$ is the vertical component of the diffusion flux density; primes denote deviations from the mean, overbars denote (ensemble) averages. ρ_0 is a reference density and c the specific heat. I stands for the penetrating component of solar radiation: $I = I_0 e^{\gamma z}$, with $I_0 (< 0)$ the surface flux of penetrating solar radiation and γ an extinction coefficient, ranging from $0.3 m^{-1}$ in polluted Dutch lakes to $0.03 m^{-1}$ in certain areas in the ocean.

With approximately homogeneous turbulence in horizontal planes and neglecting the production of turbulence kinetic energy by the mean flow shear the steady state equation for turbulent mechanical energy is (cf. Phillips, 1977)

$$\frac{\partial}{\partial z}\left\{\overline{w'\left(\frac{p'}{\rho_0} + \frac{1}{2}(u'^2 + v'^2 + w'^2)\right)}\right\} = \alpha g\overline{w'T'} - \varepsilon . \qquad (2)$$

It expresses a balance between the divergence of the turbulent vertical energy flux, the rate of working by the buoyancy force and the dissipation (ε). α is the thermal expansion coefficient and p' denotes the pressure fluctuations.

The assumption that the mean temperature is uniform over the depth of the mixed layer makes it easy to integrate the governing equations in the vertical over the mixed layer. For the one-dimensional case this was for instance worked out and discussed in detail by Niiler and Kraus (1977). A more dimensional extension was systematically derived by De Szoeke (1980). Except for the addition of advective terms in the heat transport equation an important difference with the one-dimensional models is that now the motion of the interface between the mixed layer and the interior can also be controlled by up- or downwelling (vertical advection). The resulting vertically integrated form of the heat transport equation, (1), is:

$$h \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} + [T - T_i]w_e = -\overline{w'T'}(0) - \frac{I_0}{\rho c} \tag{3}$$

where h is the mixed layer depth,

$(U,V) = \int_{-h}^{0} (u,v)dz$, the horizontal transport

$\overline{w'T'}(0)$ the surface heat flux and

T_i the temperature immediately below the mixed layer.

The entrainment velocity w_e is given by the relation

$$w_e = \frac{\partial h}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \tag{4}$$

expressing that in the case of entrainment, i.e. $w_e > 0$, the rate of change of the mixed layer depth is a balance between entrainment and the divergence of the transport in the mixed layer. In the detraining case $(w_e < 0)$ the mixed layer is decoupled from the interior as there is no temperature jump at the base of the mixed layer.

Vertical integration of (2), neglecting the flux of mechanical energy at the base of the mixed layer, gives the mechanical energy balance:

$$-mu_*^3 = \alpha g \int_{-h}^{0} \overline{w'T'}dz - \int_{-h}^{0} \varepsilon dz$$

where $u_* \equiv |\vec{\tau}/\rho_0|^{1/2}$ is the friction velocity (with $\vec{\tau}$ the surface wind stress) and m a "mixing efficiency" parameter. The mechanical energy balance states that that portion of the kinetic energy input by the wind that is not dissipated is used to increase the potential energy of the water column. It is in essence the same balance as the one proposed by Kraus and Turner (1967). In the sequel the simplifying assumption will be made that the effect of the dissipation is reflected in the value of the mixing efficiency parameter m (see also Niiler and Kraus, 1977). An expression for the convective term $\alpha g \int_{-h}^{0} \overline{w'T'}dz$ can be derived from Eq. (1). Under the "slab assumption" that the horizontal velocity is uniform over the mixed layer depth the resulting energy equation is:

$$\frac{h}{2}\left\{h\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y}\right\} = -\frac{m}{\alpha g}u_*^3 - h\overline{w'T'}(0) - \frac{I_0}{\rho c}\left(h - \frac{1}{\gamma}\right) . \tag{5}$$

Instead of merely prescribing the net downward heat flux at the surface it will be related to the difference between the mixed layer temperature and an apparent atmospheric equilibrium temperature T_A:

$$Q \equiv \overline{w'T'}(0) + \frac{I_0}{\rho c} = \lambda(T - T_A) \tag{6}$$

where T_A includes the effects of evaporation and solar radiation. The strength of the coupling between the atmosphere and the mixed layer is expressed in the value of the factor λ. Based on a heat budget analysis for zonally and time averaged conditions Haney (1971) has shown that it is of the order of $8 \times 10^{-6} ms^{-1}$.

Imposing a heat flux in the form (6) results in an extra nonlinearity in the right hand side of the energy balance (5). However, if we try to avoid that difficulty by applying just a given Q it turns out that we throw away the frontogenetic mechanism. This will be shown in the next section.

The dynamics in the mixed layer can for instance be described by a simple Ekman balance. For the cases considered here the mixed layer thicknesses will be small enough so that the geostrophic contribution to the mixed layer transport can be neglected. Therefore:

$$U = \frac{\tau^y}{\rho_0 f} \tag{7}$$

$$V = \frac{-\tau^x}{\rho_0 f} \tag{8}$$

where τ^x, τ^y are the eastward and northward components of the surface wind stress, respectively, and f is the Coriolis parameter.

3. Generation of fronts in the mixed layer

In the case of convergence of the transport in the mixed layer, with associated downwelling, and under steady state conditions Eq. (3) reduces to a linear balance between local atmospheric heating (or cooling) and horizontal advective cooling (heating). The problem can then be solved without knowledge of the oceanic interior. Also the mixed layer depth disappears from the heat balance and doesn't need to be resolved. This is the model studied by Welander (1981). He showed that if the mixed layer is very weakly coupled to the atmosphere a discontinuity in the temperature field is generated at the location of maximum downwelling. The condition of extremely weak atmosphere - mixed layer coupling (i.e. λ very small in (6)) is essential so that the strong gradients that result from the advection toward the convergence region are not wiped out by the local atmospheric heating.

De Szoeke (1980) also studied frontogenesis due to convergence of Ekman transports in the mixed layer. His model is time dependent and concentrates on the interplay between wind driven entrainment and up- or downwelling. There is no surface buoyancy flux so the dissipative effect through local atmospheric heating (or cooling) is fully absent in his model. For variations on a seasonal time scale the governing equations follow from (3) through (8) by taking $\lambda = 0$. Cushman-Roisin (1981) slightly extended De Szoeke's analysis by allowing water masses to escape laterally out of the convergence regions (confluence). Also in that study the surface buoyancy flux is neglected. However, it is well known that in almost all cases the heat flux at the air-water interface is one of the dominant factors that control the mixed layer (see for instance Gill and Niiler, 1972) and it can certainly not be neglected. In fact, this is one of the main reasons for the rather successful use of one dimensional mixed layer models in simulations of variations on the diurnal and seasonal time scale of the temperature field in the open ocean or in lakes (Niiler and Kraus (1977), Harleman (1982)).

In the remainder of this study the important role that can be played by the surface buoyancy flux in both the generation and the dissipation of fronts shall become obvious. To insolate the frontogenetic mechanism in its purest form we will apply a wind stress that is spatially uniform. Divergence or convergence of Ekman transport is therefore absent.

Also the apparent atmospheric temperature T_A will be uniform in space. It will be monotonically increasing as a function of time, indicating that we are considering the heating part of the seasonal cycle, when the seasonal thermocline is formed. Under these conditions (3) through (6) simplify to:

$$h = \frac{2mu_*^3}{\alpha g\lambda(T_A - T)} \tag{9}$$

$$\frac{\partial T}{\partial t} + \frac{\alpha g\lambda(T_A - T)}{2mu_*^3}\left(U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y}\right) = \frac{\alpha g\lambda^2}{2mu_*^3}(T_A - T)^2 \tag{10}$$

where U and V can for instance be given by (7) and (8). In calculating solutions of an initial or an initial-boundary value problem of (10) it must be remembered that it is *only* valid if $\partial h/\partial t < 0$. This condition can easily be checked through (9) and, with $T_A > T$, it is equivalent to:

$$3\frac{\partial u_*}{\partial t} - \frac{u_*}{T_A - T}\frac{\partial(T_A - T)}{\partial t} < 0 \; .$$

It shows that (in this model) detrainment can result if there is a sufficient relaxation of the wind stress or if the surface buoyancy flux increases at a fast enough rate.

Eq. (10) is a quasilinear hyperbolic partial differential equation and can be solved by the method of characteristics (e.g. Courant and Friedrichs (1948)). The effect of the nonlinearity on the left hand side can most easily be seen if we apply it to a case with no meridional transport ($V = 0$). Eq. (10) can then be interpreted as the ordinary differential equation (of Ricatti-type):

$$\frac{dT}{dt} = \frac{\alpha g\lambda^2}{2mu_*^3}(T_A - T)^2 \tag{11}$$

along characteristics that are the solution curves of:

$$\frac{dx}{dt} = \frac{U\alpha g\lambda}{2mu_*^3}(T_A - T) \; . \tag{12}$$

Let an initial temperature distribution be given by

$$T(x,0) = T_0(x) . \tag{13}$$

Different values of T_0 propagate with different velocities dx/dt as given by (12). The simplest case, with T_A spatially uniform and U and u_* constants, shows that this propagation speed is largest in regions with the coolest mixed layer water, for instance near a coastal upwelling region. In this latter important application the propagation is directed offshore, with the largest speed near the coast, decreasing toward the interior. This results in a compression of that part of the initial profile and under certain conditions the temperature gradient can sharpen so strongly that a temperature discontinuity results: a front has been generated. It is this nonlinearity in the advective term that can lead to such a steepening of the gradient with possible frontogenesis. However, whether a front will be generated is also dependent on the damping effect that is provided by the local heating, represented in the equations by the right hand side of (11). The heating acts most efficient in the coolest regions thus working to wipe out an initial horizontal temperature gradient. These two processes: local heating and differential propagation of the initial profile are therefore competitive.

To investigate if and under what conditions fronts may form we shall solve (11) through (13) for a T_A that increases linearly in time:

$$T_A = a + bt \quad (b > 0) .$$

The transport U and the shear velocity u_* are taken constant. Under these conditions the solution is:

$$T(t,\xi) = a + bt + \frac{bp}{\lambda}\left(\frac{1 + \mu(\xi)e^{2\lambda pt}}{1 - \mu(\xi)e^{2\lambda pt}}\right) \tag{14}$$

along the characteristic curve (through the point $(\xi,0)$ on the initial line) that has the equation:

$$x = \xi - Up\left\{t + \frac{1}{\lambda p} \ln\left|\frac{1 - \mu(\xi)}{1 - \mu(\xi)e^{2\lambda pt}}\right|\right\} \tag{15}$$

where

$$\mu(\xi) = \frac{T_0(\xi) - a - b/\lambda p}{T_0(\xi) - a + b/\lambda p} \tag{16}$$

$$p = \left(\frac{\alpha g b}{2mu_*^3}\right)^{1/2}$$

and ξ acts as a parameter.

The family of characteristics given by (15) forms an envelope for:

$$t = \frac{1}{2\lambda p} \ln\left\{\frac{U\mu' + \lambda(1 - \mu)}{U\mu' + \lambda\mu(1 - \mu)}\right\} \tag{17}$$

(' denotes differentiation with respect to ξ). Such an envelope forms the boundary of a region in the (x,t)-plane where the characteristics overlap leading to multivaluedness of the solution. If an envelope is formed then it determines the position of points at which $\partial T/\partial x$ will first become infinite and a front forms. It is easy to show that the condition that this happens for a $t > 0$ is equivalent to

$$UT_0' > \lambda(T_A(0) - T_0) + \left(\frac{2mu_*^3 b}{\alpha g}\right)^{1/2} \tag{18}$$

expressing the fact that the initial temperature gradient in the mixed layer must be large enough so that the damping effect by the local heating doesn't prevent the temperature wave to "break". Obviously UT_0' has to be positive, which means that the cooler water must be upstream. The inequality (18) also shows that having the largest initial temperture gradient in the region with the smallest temperature difference with the atmosphere provides the most favorable condition for a temperature discontinuity to develop. A weak coupling to the atmosphere (λ small) also seems favorable, but (17) shows that the smaller λ the later the front is formed.

Once a front has been established it propagates with a velocity that can be determined from the mass conservation law in integral form. This is worked out in the appendix. The result is that

$$\frac{d\ell}{dt} = \frac{U}{h_2} \tag{19}$$

where $\ell(t)$ is the location of the front at time t and h_2 is the depth of the mixed layer immediately behind the front. It expresses that, in this model, the shallower layer behind the front simply overtakes the deeper mixed layer ahead of it without being slowed down.

4. Results and discussion

In the preceding sections we have shown that if condition (18) is satisfied an initially smooth horizontal temperature profile in the mixed layer will develop into a front. The frontogenesis is not a result of divergent or convergent wind or temperature fields: in the analysis the imposed wind stress and atmospheric temperature fields at the surface are both uniform in the spatial coordinate. The essential feature is that the surface buoyancy flux is not just an imposed quantity but that it is related to the temperature difference between the mixed layer and the atmosphere. In this way the initial horizontal temperature gradient in the mixed layer, combined with a spatially uniform atmospheric temperature, gives a nonuniform surface buoyancy flux. This in turn produces a horizontal gradient in the mixed layer depth, and, with a uniform wind-driven transport, a compression or expansion of the initial temperature "wave" results.

Before going over to a discussion of the possible structure of the front it is important to find out if the formal condition (18) can be satisfied for realistic values of the different physical parameters in the problem and if so whether an associated front is formed within a reasonable time span (as given by (17)). For that purpose we will calculate a few examples. The first one is that of a near equatorial region, say at 6°S. The applied wind stress is northward, leading to offshore Ekman transport and associated coastal upwelling. In the upwelling region itself the above formulated model is certainly much too simplified to be applicable. The focus is on the region somewhat farther offshore, where the advection of relatively cool water out of the upwelling area still maintains a horizontal density gradient. For the different parameters we take the values:

$\lambda = 8.75 \times 10^{-6} ms^{-1}$ (Haney, 1971); $m = 1.25$ (see Niiler and Kraus (1977) for a discussion on this "mixing efficiency parameter");

$\rho_0 = 10^3 kgm^{-3}$; $\alpha = 2 \times 10^{-4} °C^{-1}$; $b = 6 \times 10^{-7} °Cs^{-1}$;

$\tau^y = 10^{-1} kgm^{-1}s^{-2}$ and $T_A(0) - T_0 = 1°C$.

Substitution of these values in (18) gives the condition that if the initial horizontal temperature gradient satisfies

$$T_0' < -5.5 \times 10^{-6} °Cm^{-1}$$

then a front will result. For instance, if $T_0' = -7.5 \times 10^{-6} °Cm^{-1}$ the associated time t_F in which the temperature discontinuity is generated follows from (17) to be $t_F \approx 4.2 \times 10^6 s$ (about 48 days). If $T_0' = -10^{-5} °Cm^{-1}$ this time decreases to $2.7 \times 10^6 s$ (about 31 days). If we repeat the above calculations for a weaker wind stress: $\tau^y = 5 \times 10^{-2} kgm^{-1}s^{-2}$ the resulting condition is that $T_0' < -7.5 \times 10^{-6} °Cm^{-1}$. For instance, if $T_0' = -10^{-5} °Cm^{-1}$ (17) now gives $t_F \approx 2.5 \times 10^6 s$.

East west temperature gradients of the above calculated order of magnitude are not unusual in the near equatorial eastern Pacific. Therefore, under such conditions the frontogenetic mechanism as studied here might be a non-negligible process in the extremely complicated dynamics of that area. The larger absolute value of T_0' that is required for a weaker wind stress reflects the fact that local heating is more dominant under weak winds: the temperature wave can only "break" if its initial slope is steep enough. This notion becomes especially dramatic if we repeat the above calculations for regions in higher latitudes. With the same parameter values as given above, the associated condition for T_0' becomes unrealistic. For instance at 30°S with a $10^{-1} kgm^{-1}s^{-2}$ wind stress there results $T_0' < -27.5 \times 10^{-6} °Cm^{-1}$. This is caused by the much smaller Ekman transports in higher latitudes resulting in a dominant role of the local buoyancy flux in the dynamics of the mixed layer. For the oceanic environment the nonlinear mechanism for steepening of the horizontal temperature gradient as proposed in this study is therefore of possible importance only in near equatorial regions.

However, this doesn't mean that the phenomenon doesn't exist in middle or higher latitudes: an important field of application might be that of lakes and small seas. After a period of strong winds along the length axis of a stratified lake, with downwelling and suppression of the thermocline at the downwind end and upwelling at the upwind side of it, it is not unusual that in the upper layer a horizontal temperature gradient results of the order of $10^{-4} °Cm^{-1}$. If we then apply a wind stress of $5 \times 10^{-2} kgm^{-1}s^{-2}$ and take the other parameter values as earlier, a front develops in the order of two weeks. Over such a time period for a lake of say 40 km length and with the initial maximum temperature gradient close to the upwind shore, the jumps in temperature and mixed layer depth appear near the middle of the lake after which the front travels toward the downwind boundary. Such a development is pictured in Fig. 1, showing qualitatively the same evolution as in the oceanic case, but with the smaller horizontal length scale of the lake leading to a shorter time of frontogenesis.

Now that we have shown that the frontogenetic mechanism acts on realistic time and length scales for lakes and for the near equatorial regions of the ocean it becomes relevant to address the problem of finding the structure of such fronts. What we have constructed in the preceding section is a correct "weak solution" of (10) with initial condition (13), where for the unique determination of the jump discontinuity in the mixed layer temperature (and associated depth) we had to return to the original mass conservation law (for a discussion on this and related topics see Whitham, 1974). The question is now what physical process can be expected to lead to a smoothing of that step structure to a continuous profile of which the discontinuous weak solution is a first approximation. One obvious candidate that was not incorporated in the model formulation is horizontal turbulent diffusion. If we include that process in (1) with a constant eddy diffusion coefficient K and repeat all the manipulations that led to (10) an extra term $K(\partial^2T/\partial x^2 + \partial^2T/\partial y^2)$ must be added to the right hand side of that equation. In order to get an idea of the possible importance of such eddy diffusion the equation will be scaled. For that let nondimensional variables (denoted by primes) be defined by:

$$T = T_r + \vartheta T'; \quad T_A = a + \vartheta T_A';$$

$$x = Lx'; \quad t = \frac{2mu_*^3 L}{\alpha g \lambda \vartheta |U|} t' .$$

The resulting nondimensional version of Eq. (10) with horizontal diffusion added (and omitting the primes) is:

$$\frac{\partial T}{\partial t} + \mathrm{sgn}(U)(T_A - T)\frac{\partial T}{\partial x} = \mu(T_A - T)^2 + \epsilon\frac{\partial^2 T}{\partial x^2} \tag{20}$$

where the parameters μ and ϵ are defined by:

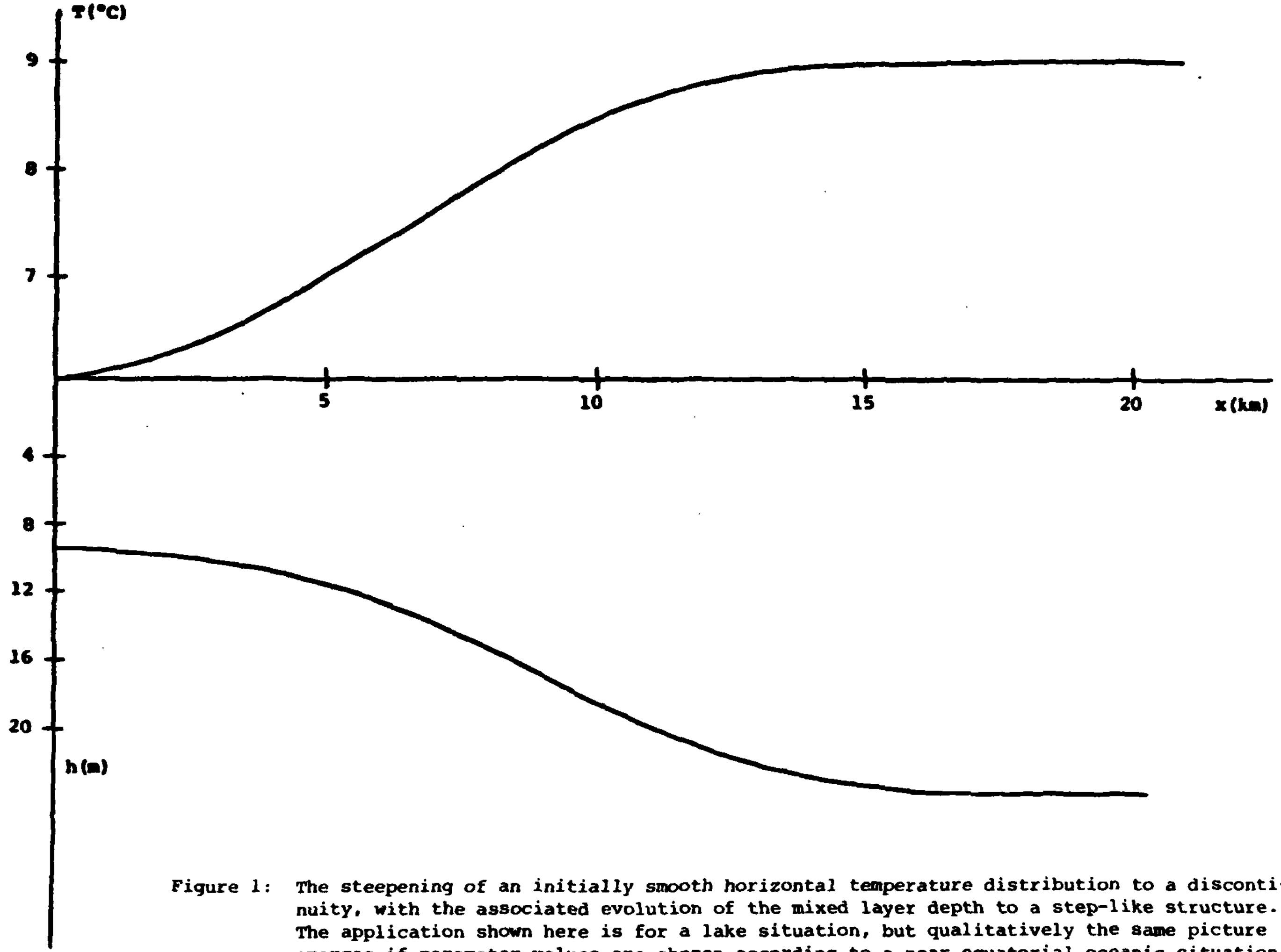

Figure 1: The steepening of an initially smooth horizontal temperature distribution to a discontinuity, with the associated evolution of the mixed layer depth to a step-like structure. The application shown here is for a lake situation, but qualitatively the same picture emerges if parameter values are chosen according to a near equatorial oceanic situation. For this figure the parameter values are: $a = 11°C$; $b = 10^{-6}°Cs^{-1}$; $\lambda = 7.5 \times 10^{-6}ms^{-1}$; $U = 0.45m^2s^{-1}$; $u_* = 7 \times 10^{-3}ms^{-1}$; $m = 1.25$.

(a) $t = 0$: the initial profiles of mixed layer temperature and depth.

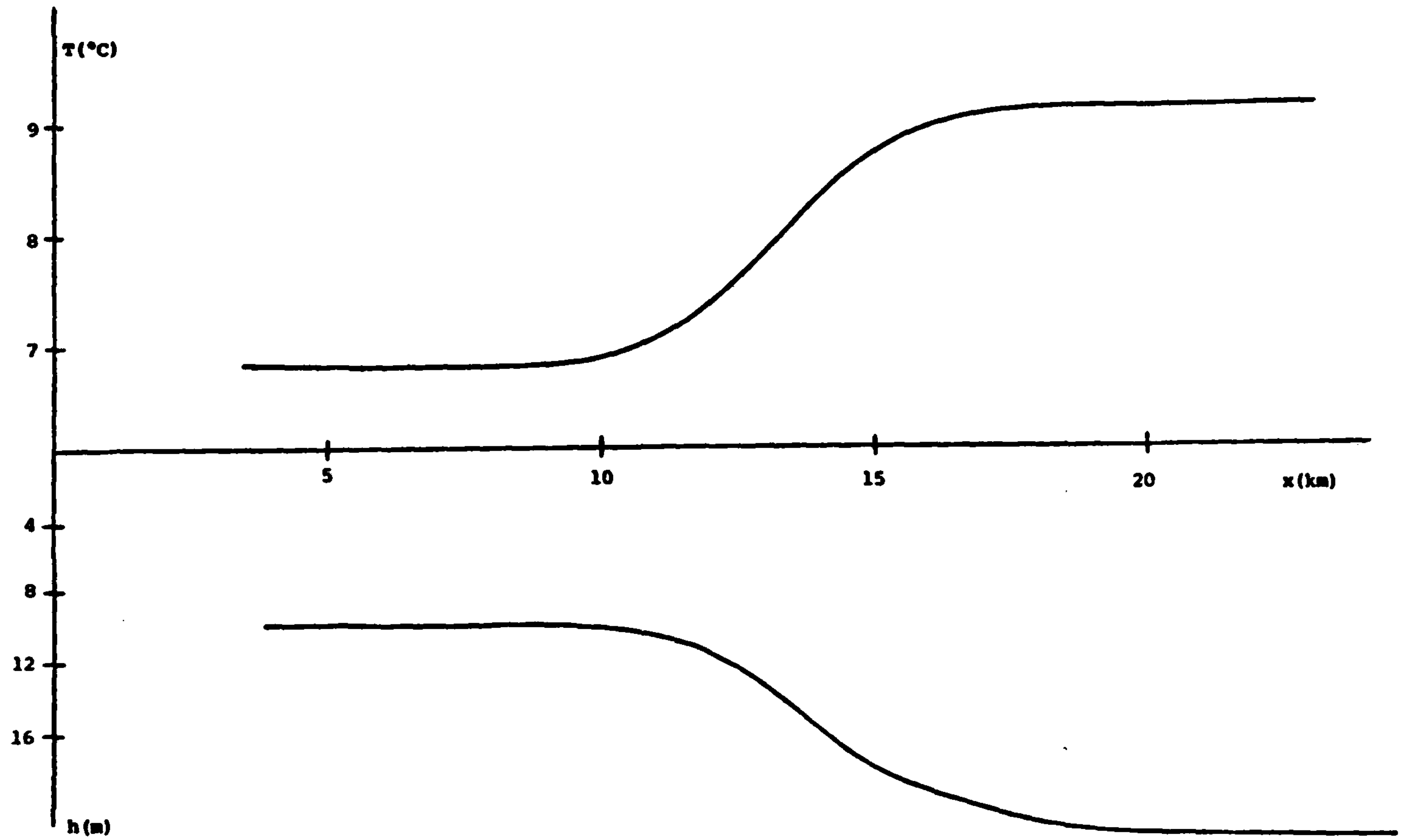

Figure 1: (b) $t = 2.5 \times 10^5$s: the evolution after approximately 3 days.

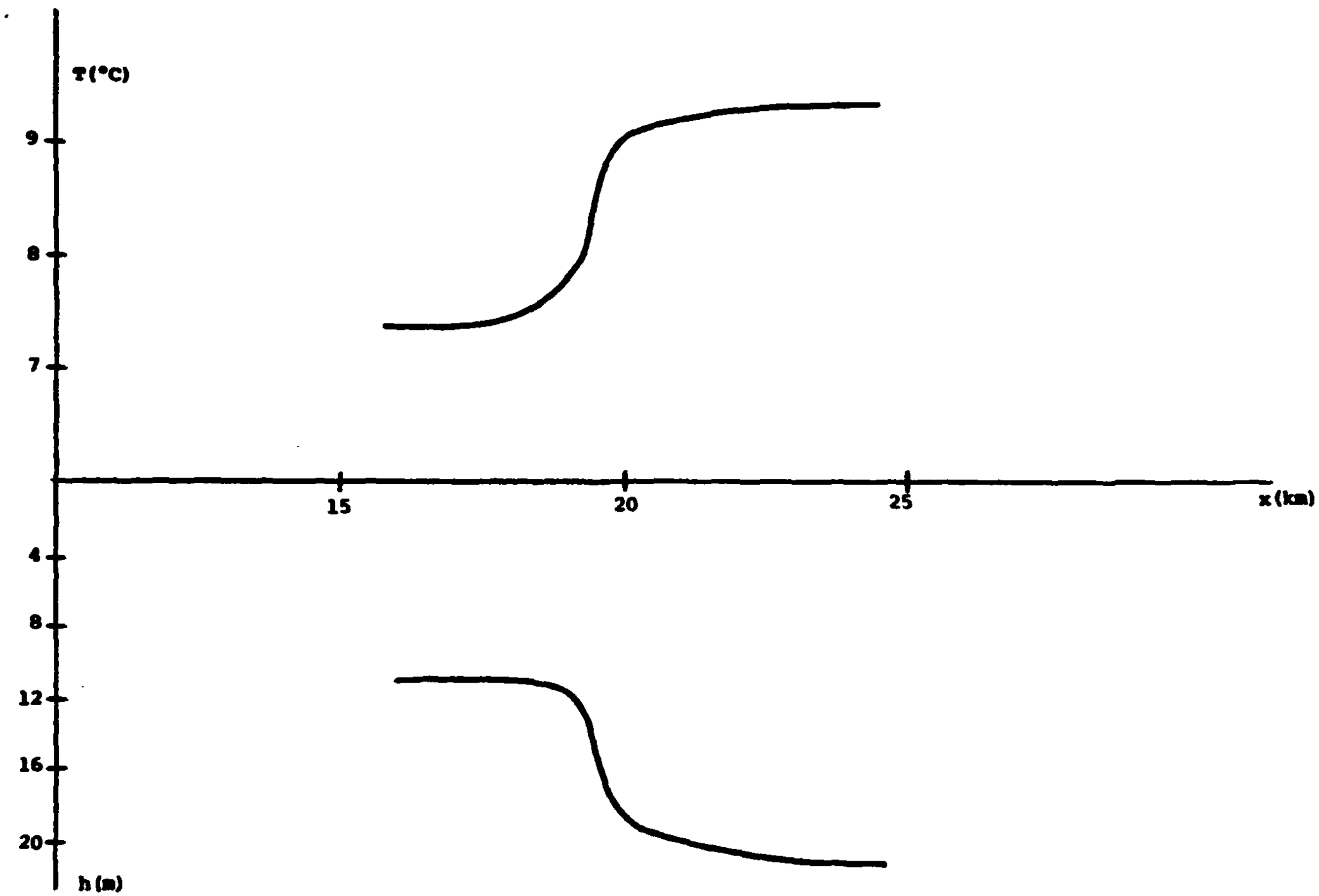

Figure 1: (c) $t = 5 \times 10^5$s: just before a front is formed.

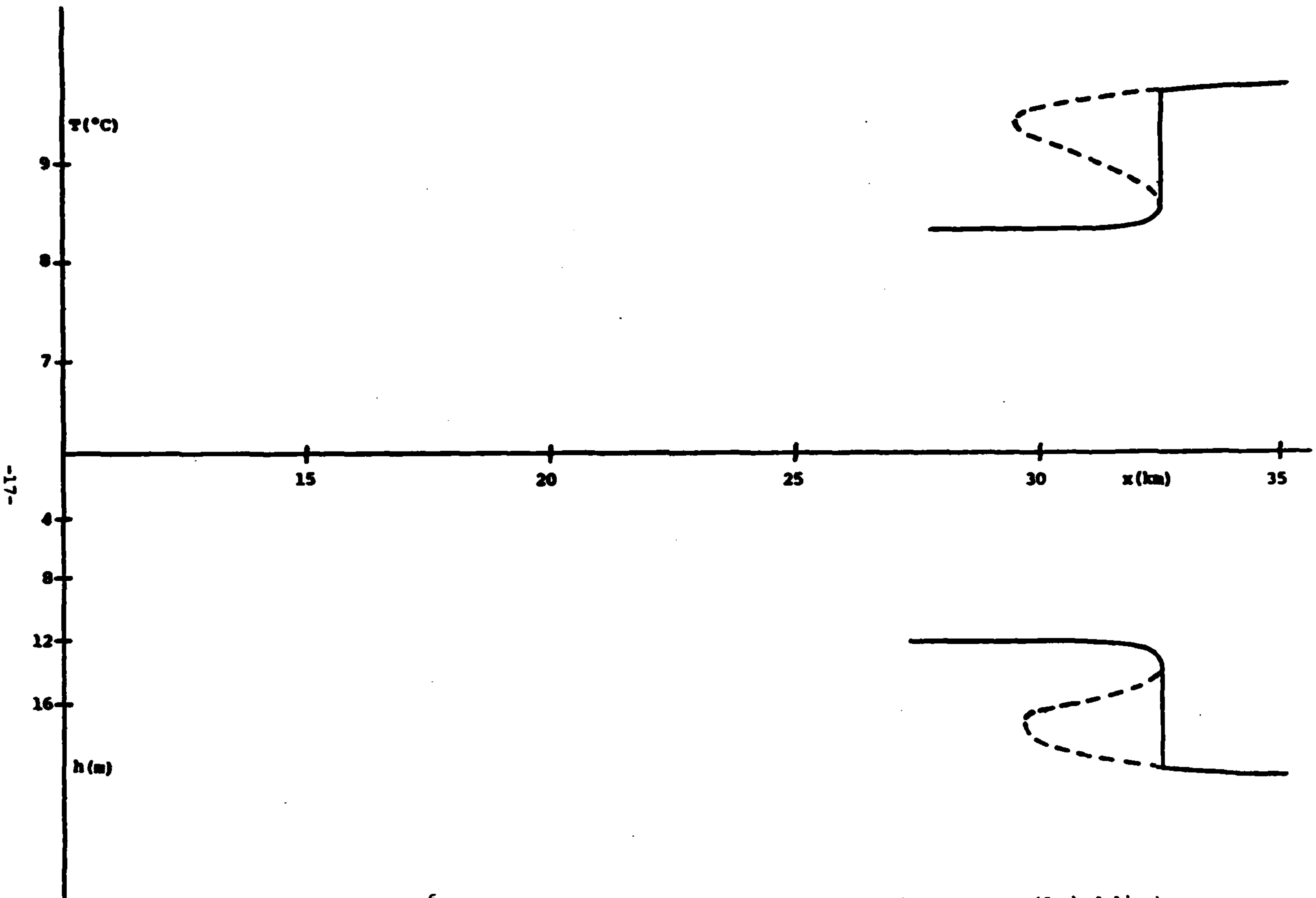

Figure 1: (d) $t = 10^6$ s: the step like structure replaces the multivalued part (dashed line) of the solution (14) and (15).

$$\mu = \frac{\lambda L}{|U|}, \quad \epsilon = \frac{K2mu_*^3}{\alpha g \lambda \vartheta |U| L}.$$

μ measures the relative importance of local heating to the advective heat flux, ϵ that of diffusion to advection. Characteristic values for the lake example are: $L = 3 \times 10^4 m$; $|U| = 5 \times 10^{-1} m^2 s^{-1}$; $u_* = 7 \times 10^{-3} ms^{-1}$; $\vartheta = 3°C$; $K = 10 m^2 s^{-1}$. These lead to the values: $\mu = 0.5$ and $\epsilon = 1.1 \times 10^{-2}$. Consequently (20) is a singular perturbation problem, and only in regions with very strong temperature gradients the diffusion becomes a nonnegligible part of the balance. In this way the front as a discontinuity in the temperature distribution is replaced by a diffusive boundary layer structure. Eq. (20) suggests a dimensional measure for the boundary layer width to be ϵL, which is $K2mu_*^3/(\alpha g \lambda \vartheta |U|)$. Substituting the above given values for the scale factors gives an order 300m width. If we repeat the example for the near equatorial ocean, with $L = 10^6 m$ and $U = 3.3 m^2 s^{-1}$ (corresponding to a $5 \times 10^{-2} kgm^{-1} s^{-2}$ wind stress) we find the values: $\mu = 2.7$ (indicating that local heating is relatively more important in those regions) and $\epsilon = 5 \times 10^{-5}$ so that now the boundary layer is considerably narrower: O(50m).

Another process that can be expected to become active when the gradient steepens is free convection as a result of gravitational instability. It is at this point that the dynamics in the region below the mixed layer becomes relevant also for the case of a shallowing mixed layer. When the mixed layer shallows it leaves a "trace" behind in the interior. Suppose for example that the interior dynamics are such that this trace is approximately conserved. When the front passes it replaces the existing mixed layer by a much shallower and cooler one, thus creating a gravitationally instable situation. The resulting overturning with associated vigorous mixing can effectively wipe out the discontinuities and replace it by continuous profiles. In this way the effect of the instability appears to be horizontally diffusive and a naive way of parameterizing it might be through an apparent eddy diffusion term as in (20). It is clear that the "eddy diffusivity" K is in this case, among other things, strongly dependent on the dynamics in the interior in relation to the transport characteristics in the mixed layer. Both in lakes (see for example Imberger and Hamblin, 1982) and in the (near) equatorial ocean (e.g. Moore and Philander, 1977) there can be considerable dynamic activity below the mixed layer. Associated values for K will therefore differ substantially from one situation to the other.

When the gradients of temperature and mixed layer depth become steeper the associated cross frontal pressure gradient also increases. As a result a baroclinic jet along the front will develop. Except for its possible role in maintaining the front (Cushman-Roisin, 1981) it is also interesting to observe that in the near equatorial case such a jet will be equatorward so that in this way water from the coastal upwelling region, while being intensely warmed, is transported into the equatorial zone.

Of course, such effects can only be studied quantitatively through much less simplified models than the one we have presented here. The purpose of this study has been to identify through the simplest possible model one of the physical mechanisms that can be responsible for the formation of fronts in the upper mixed layer of lakes or the near equatorial ocean.

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APPENDIX

The propagation of the front.

We start with the mass (or buoyancy) conservation law in time integrated form:

$$\int_{x_1}^{x_2}\int_{-h(x,t)}^{0} (\rho(x,t) - \rho^0(x,t_0))dzdx = \int_{t_0}^{t}\int_{x_1}^{x_2} Bdxdt -$$

$$- \int_{t_0}^{t} \{\int_{-h(x_2,t)}^{0} u(x_2,z,\tau)\rho(x_2,\tau)dz - \int_{-h(x_1,t)}^{0} u(x_1,z,\tau)\rho(x_1,\tau)dz\}d\tau \qquad \text{(A1)}$$

stating that, in the absence of vertical advection and for a decreasing mixed layer depth over the time interval (t_0,t), the mass in the column $(x_1,x_2) \times (-h(x,t),0)$ changes as a result of the total buoyancy flux at the surface over that time interval (first term on right side) and the net inflow of mass across the boundaries in x_1 and x_2.

Let the position of the front be at $x = \ell(t)$, and take $x_1 < \ell(t) < \ell(t_0) < x_2$ (corresponding to "westward" advection: $u < 0$). Let the mixed layer be of "slab type" and the horizontal transport U constant, so that $u(x_i,z,\tau) = U/h(x_i,\tau)$ for $-h(x_i,t) < z < 0$. To derive the front velocity $d\ell/dt$ from (A1) we differentiate first with respect to t, giving:

$$h(\ell^-,t)(\rho(\ell^-,t)-\rho^0(\ell^-,t_0))\frac{d\ell}{dt} + \int_{x_1}^{\ell(t)} \{\frac{\partial h}{\partial t}(x,t)(\rho(x,t)-\rho^0(x,t_0))+h\frac{\partial\rho}{\partial t}(x,t)\}dx$$

$$- h(\ell^+,t)(\rho(\ell^+,t)-\rho^0(\ell^+,t_0))\frac{d\ell}{dt} + \int_{\ell(t)}^{\ell(t_0)} \{\frac{\partial h}{\partial t}(\rho(x,t)-\rho^0(x,t_0))+h\frac{\partial\rho}{\partial t}\}dx$$

$$+ \int_{\ell(t_0)}^{x_2} \{\frac{\partial h}{\partial t}(\rho(x,t)-\rho^0(x,t_0))+h\frac{\partial\rho}{\partial t}\}dx = \int_{x_1}^{x_2} Bdx-U(\rho(x_2,t)-\rho(x_1,t)) \qquad \text{(A2)}$$

where

$$h(\ell^-,t)(\rho(\ell^-,t) - \rho^0(\ell^-,t_0)) = \lim_{\substack{x\to\ell(t)\\ x>\ell(t)}} h(x,t)(\rho(x,t) - \rho^0(x,t_0))$$

and

$$h(\ell^+,t)(\rho(\ell^+,t) - \rho^0(\ell^+,t_0)) = \lim_{\substack{x\to\ell(t)\\ x<\ell(t)}} h(x,t)(\rho(x,t) - \rho^0(x,t_0)) .$$

For $t_0 \to t(t_0 < t)$ we have

$$\rho^0(\ell^-,t_0) \to \rho(\ell^-,t)$$

and

$$\rho^0(\ell^+,t_0) \to \rho(\ell^-,t) ,$$

which can easily be understood from Fig. A1 to result from the fact that $\rho^0(\ell^-,t_0) = \rho^0(\ell^+,t_0)$. If we let first $t_0 \to t$ in (A2), using the above relations, and then $x_1,x_2 \to x$, using the fact that $h\partial\rho/\partial t$ is bounded on (x_1,ℓ) and (ℓ,x_2), Eq. (A2) reduces to:

$$h(\ell^+,t)(\rho(\ell^+,t) - \rho(\ell^-,t)) \frac{d\ell}{dt} = U(\rho(\ell^+,t) - \rho(\ell^-,t))$$

which finally gives the propagation velocity of the front as:

$$\frac{d\ell}{dt} = \frac{U}{h(\ell^+,t)} . \tag{A3}$$

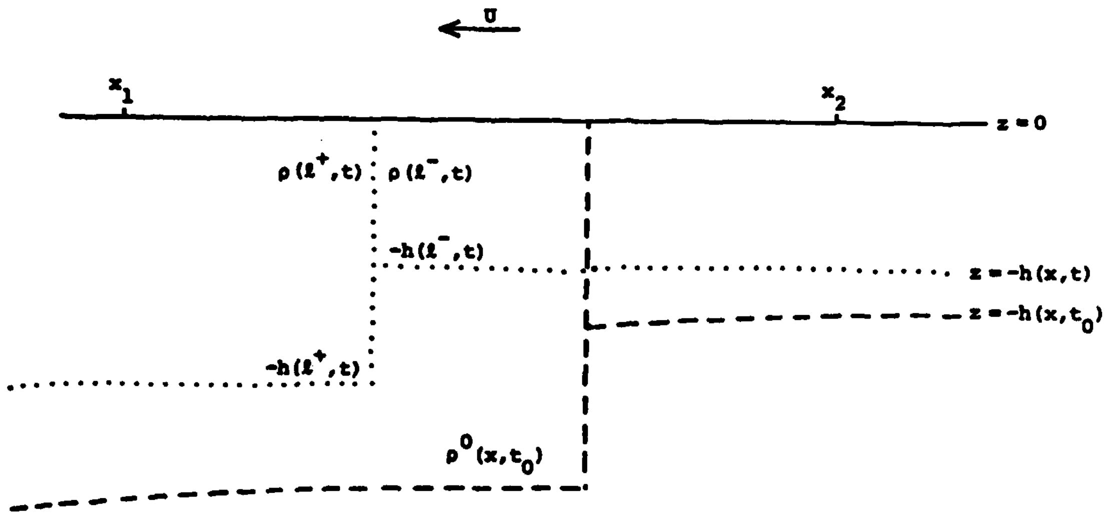

Figure A1: The propagation of the front in the mixed layer: as time evolves from t_0 to t the front propagates from $\ell(t_0)$ to $\ell(t)$ with the mixed layer becoming shallower on both sides of the front. The dashed line marks the lower boundary $z = -h(x,t_0)$ of the mixed layer at time t_0, the dotted line that at time t.

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Through an analysis of a more dimensional extension of the bulk mixed layer model of Kraus and Turner (1967) it is shown how even under spatially uniform atmospheric conditions an initially smooth horizontal temperature gradient in the surface mixed layer can still develop into a front. Essential for it is the notion that the net downward heat flux at the air-water interface is related to the difference between the mixed layer temperature T and an apparent atmospheric temperature T_A (Haney, 1971) so that the initial gradient in T

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20. ABSTRACT - cont'd.

corresponds to a horizontal variation in the surface buoyancy flux. As a result also the mixed layer depth differs from place to place. Depending on the direction of the wind driven transport this produces either a steepening or a flattening of the initial temperature profile. A lower bound condition for the initial horizontal advective heat flux is derived in terms of the initial surface heat flux, the stirring by the wind and the time rate of change of the apparent atmospheric temperature. If that condition is satisfied a front develops. If not, then frontogenesis is prevented by the damping effect of the local atmospheric heating. It is shown that the condition can be satisfied on the scales of lakes (or small seas) and in near equatorial oceanic regions. Mathematically the problem boils down to a first order quasilinear hyperbolic partial differential equation that is solved exactly by the method of characteristics.